There are four flight conditions selected out of the possible range, whose parameters are given in Table 1.

According to Refs. 4 and 5, a dynamic compensator of order 2 guarantees exact pole assignment for one flight condition. Therefore, we try to stabilize all four flight conditions with a second-order dynamic compensator.

Numerically, we compute the matrix K of Eq. (8) that minimizes Eq. (11) for the augmented system (10) and obtain

$$K = \begin{bmatrix} 0.0288 & 0.0191 & 0.0127 \\ -3.7633 & -4.2102 & 3.0319 \\ 4.7533 & -0.4610 & -3.5571 \end{bmatrix}$$

From Eq. (9), the corresponding control is

$$u(t) = (0.0191 \;\; 0.0127)z(t) + 0.0288y(t)$$

where

$$\dot{z}(t) = \begin{bmatrix} -4.2102 & 3.0319 \\ -0.4610 & -3.5571 \end{bmatrix} z(t) + \begin{bmatrix} -3.7633 \\ 4.7533 \end{bmatrix} y(t)$$

This control stabilizes all four flight conditions.

Although the proposed numerical procedure for solving the simultaneous stabilization problem will not always lead to a solution, this example illustrates that it does offer a possible method and can be included in our toolbox of techniques.

VI. Conclusions

A numerical procedure for determining a dynamic output feedback controller was presented. Such a controller has the advantage of being of lower order than a controller based on the separation theorem where an estimate of the entire state is used. It can also produce a stable system when static output feedback fails. The procedure was extended to the problem of the simultaneous stabilization of several plants by a single controller.

Acknowledgment

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Algebraic Approach to the Bearings-Only Estimation Equations

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Introduction

THE problem of interest is that of locating a target from line-of-sight bearing measurements performed from a moving platform. Most recursive algorithms implement some variant of the extended Kalman filter (EKF).[1-3] Common to the EKFs thus far described in the literature is that the measurement equation yields a bearing angle relative to some known axis. The filter equations, as a consequence, are expressed in terms of the transcendental arctangent function. Though mathematically correct, the need to calculate the arctangent function imposes demands on the sample period of a real-time estimation system, especially if a numerical integration or other type of iteration is required. Furthermore, the extension of the equations to the three-dimensional problem is complicated, and consideration of more than one measurement array axis is cumbersome.

In this Note, two examples of filter equations are derived for which the measurement equation yields the directional cosine of the target relative to a known axis. (The directional cosine measurement is the natural output of many angle sensors and is often unnecessarily converted to angle by applying the arccosine function.) This redefinition of the measurement equation leads to filter equations that are nonlinear algebraic rather than transcendental. These equations are, furthermore, inherently three-dimensional. Inclusion of multiple measurement axes is straightforward.

Algebraic Filter Equations

To demonstrate the utility of these algebraic derivations to bearings-only estimation, two EKFs are presented. The two EKFs differ in their state variable assignment. The first filter to be derived has a linear measurement equation and a nonlinear state propagation equation. The second filter has a nonlinear measurement equation and linear state propagation equation. The following symbols are used in the derivations: $\boldsymbol{R}$ is the unknown line of sight vector from the platform to the target; $|\boldsymbol{R}|$ the unknown range from the platform to the target; $\boldsymbol{r}$ the unknown line-of-sight unit vector, $= \boldsymbol{R}/|\boldsymbol{R}|$; $\boldsymbol{V}$ the known velocity vector of the platform; and $\boldsymbol{a}$ the known unit vector along the measurement axis.

Case 1: Nonlinear-in-State, Linear Measurement Extended Kalman Filter

Noting that the three components of $\boldsymbol{r}$ are not independent, the (pseudo) state vector is defined as

$$\boldsymbol{x} = \begin{bmatrix} |\boldsymbol{R}| \\ \boldsymbol{r} \end{bmatrix}$$

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To derive the plant equation, we consider each component of x separately. The quantity $d|R|/dt$ is the platform-target range rate and, thus, is equal to the negative component of the the platform velocity in the direction of the line of sight.

$$\frac{d}{dt}|R| = -r^T V \tag{1}$$

The quantity dr/dt is derived as follows

$$\frac{d}{dt} r = \frac{d}{dt}\left(\frac{R}{|R|}\right) = \frac{\dot{R}}{|R|} - \frac{R}{|R|^2}\frac{d}{dt}|R| \tag{2}$$

Noting that $\dot{R} = -V$ and substituting Eq. (1) into Eq. (2), the equation for $\dot{r}$ is given as

$$\frac{d}{dt} r = -\frac{V}{|R|} + \frac{R}{|R|^2} r^T V = -\frac{V}{|R|} - \frac{r}{|R|} r^T V$$

$$\frac{d}{dt} r = -\frac{1}{|R|}(I - rr^T)V \tag{3}$$

The plant equation for this filter is thus given by

$$\dot{x} = \frac{d}{dt}\begin{bmatrix} |R| \\ r \end{bmatrix} = \begin{bmatrix} -r^T V \\ -\frac{1}{|R|}(I - rr^T)V \end{bmatrix} = f(x)$$

The matrix of partial derivatives $\partial f/\partial x$ required for propagating the state covariance is derived as follows

$$F = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial(-r^T V)}{\partial |R|} & \frac{\partial(-r^T V)}{\partial r} \\ \frac{\partial[-(1/|R|)(I - rr^T)V]}{\partial |R|} & \frac{\partial[-(1/|R|)(I - rr^T)V]}{\partial r} \end{bmatrix}$$

The elements of F are given as

$$F_{11} = \frac{\partial(-r^T V)}{\partial |R|} = 0$$

$$F_{12} = \frac{\partial(-r^T V)}{\partial r} = -V^T$$

$$F_{21} = \frac{\partial[-(1/|R|)(I - rr^T)V]}{\partial |R|} = -\frac{\partial(1/|R|)}{\partial |R|}(I - rr^T)V$$

$$= \frac{1}{|R|^2}(I - rr^T)V$$

$$F_{22} = \frac{\partial[-(1/|R|)(I - rr^T)V]}{\partial r} = -\frac{1}{|R|}\frac{\partial[(I - rr^T)V]}{\partial r}$$

$$= \frac{1}{|R|}\frac{\partial(rr^T)V}{\partial r} = \frac{1}{|R|}\left(\frac{\partial r}{\partial r} r^T V + r\frac{\partial r^T V}{\partial r}\right)$$

$$= \frac{1}{|R|}[I(r^T V) + rV^T] = \frac{r^T V}{|R|}\left(I + \frac{rV^T}{r^T V}\right)$$

Therefore, F is given by

$$F = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & -V^T \\ \frac{1}{|R|^2}(I - rr^T)V & \frac{r^T V}{|R|}\left(I + \frac{rV^T}{r^T V}\right) \end{bmatrix}$$

The measurement equation is linear and is given by

$$y = a^T r = Cx, \qquad C = [0 \; a^T] \tag{4}$$

Case 2: Linear-in-State, Nonlinear Measurement Extended Kalman Filter

For this situation, the state vector is defined as

$$x = R = \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix}$$

The plant equation is linear and is given by

$$\dot{x} = -\dot{R} = \begin{bmatrix} -V_x \\ -V_y \\ -V_z \end{bmatrix}$$

The measurement equation is nonlinear and is given by

$$y = a^T r = \frac{1}{|R|} a^T R = \frac{1}{|x|} a^T x = C(x)$$

The observation partials $H = \partial C/\partial x$ are derived as follows

$$C(x) = \frac{1}{|x|} a^T x$$

$$\frac{\partial C(x)}{\partial x} = \frac{1}{|x|} a^T + a^T x \frac{\partial}{\partial x}\left(\frac{1}{|x|}\right)$$

$$= \frac{1}{|x|} a^T - \frac{1}{|x|^2} a^T x \frac{\partial |x|}{\partial x} \tag{5}$$

Noting that $x = R$, $|x| = |R| = R^T r$, Eqs. (5) yield

$$\frac{\partial C(x)}{\partial x} = \frac{1}{|R|} a^T - \frac{1}{|R|^2} a^T R \frac{\partial |R|}{\partial R}$$

$$= \frac{1}{|R|} a^T - \frac{1}{|R|^2} a^T R\, r^T$$

$$= \frac{1}{|R|} a^T - \frac{1}{|R|} a^T rr^T$$

$$= \frac{1}{|R|} a^T (I - rr^T) \tag{6}$$

Noting that $r = x/|x|$, Eqs. (6) can be expressed in terms of x

$$\frac{\partial C(x)}{\partial x} = \frac{1}{|x|} a^T \left(I - \frac{xx^T}{x^T x}\right) \tag{7}$$

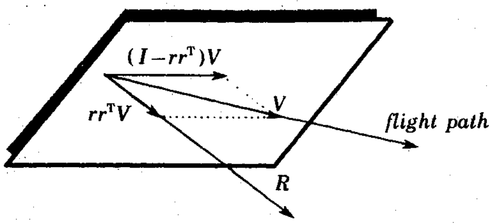

Fig. 1 Geometric interpretation of projection matrices rr^T and $I-rr^T$.

Three-Dimensional Ranging and Multiple-Axes Measurements

These equations inherently apply to the full three-dimensional, bearings-only ranging filter. Multiple measurement axes are incorporated into the derivations by letting $a = [a_1 \; a_2]$, where a_1 and a_2 are the noncolinear measurement axes.

Geometric Interpretation

Examination of the matrices rr^T and $I-rr^T$ provide insight to the fundamental geometry of the target location estimation problem. These matrices are symmetric, idempotent orthogonal projection matrices.[4] The matrix rr^T projects all vectors onto the space spanned by r. The matrix $I-rr^T$ is the orthogonal complement of rr^T and projects all vectors onto the space orthogonal to r (the null space rr^T). This relationship is readily seen by noting that $(I-rr^T)r = 0$. This geometrical significance is illustrated in Fig. 1.

The expressions rr^TV and $(I-rr^T)V$ are understood to be the radial and circumferential velocities of the platform about the target, respectively. The expression $(1/|R|)(I-rr^T)V$ represents the angular velocity of the platform about the target. The similarity of Eqs. (3) and (6) is a consequence of the fact that Eq. (3) describes the effect of the relative motion of the platform moving about the target and Eq. (6) describes the effect of the relative motion of the target moving about the platform.

Conclusions

By defining the measurement equation for a bearings-only estimation filter in terms of the directional cosine to a known axis rather than the angle to the axis, the resulting filter equations are algebraic rather than transcendental. The three-dimensional problem and the multiple-axes problem is inherently solved, and additional insight to the geometry of the problem is achieved.

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Book Announcements

VARDULAKIS, A. I. G., ***Linear Multivariable Control: Algebraic Analysis and Synthesis Methods,*** Wiley, Chichester, England, U.K., 1991, 369 pages, $125.00

Purpose: This book presents a detailed account of polynomial matrix descriptions (PMDs) of linear multivariable control systems. It is meant for graduate students and researchers.

Contents: Real rational vector spaces and rational matrices; polynomial matrix models; pole and zero structure of rational matrices at infinity; dynamics of polynomial matrix models; proper and Ω-stable rational functions and matrices; feedback system stability and stabilization; some algebraic design problems.

HABETS, L. C. G. J. M., ***Robust Stabilization in the Gap-Topology,*** Lecture Notes in Control and Information Sciences, Vol. 150, Springer-Verlag, Berlin, 1991, 126 pages, $25.00.

Purpose: This monograph deals with robust stabilization based on the notion of the gap between two closed subspaces.

Contents: Robust stabilization; the gap-topology; sufficient conditions for robust stabilization; optimally robust control and controller design; order reduction of the compensator.

AHMED, A., ***Semigroup Theory with Applications to Systems and Control,*** Longman Scientific and Technical, U.K., Copublished in the U.S. with Wiley, New York, 1991, 282 pages, $40.00.

Purpose: This monograph is an introduction to semigroup theory, a unified theory for the study of differential equations on Banach space. Applications to control and stability are treated.

Contents: Basic properties of semigroups; generation theorems for semigroups; special properties; perturbation theory for semigroups; differential equations on Banach space; stochastic differential equations; applications to systems and control.

APLEVICH, J. D., ***Implicit Linear Systems,*** Lecture Notes in Control and Information Sciences, Vol. 152, Springer-Verlag, Berlin, 1991, 176 pages, $31.00.

Purpose: This monograph treats the implicit model of linear dynamical systems and its applications to analysis and design.

Contents: System models; the Kronecker form; analysis of singularities; systems of minimal dimension; canonical representations; algebraic design applications; optimization with quadratic cost; system identification; large-scale systems; extensions.

KAMP, Y. and **HASLER, M.,** ***Recursive Neural Networks for Associative Memory,*** Wiley, Chichester, England, U.K., 1990, New York, 1991, 194 pages.

Purpose: This book discusses the different problems which arise in the analysis and design of discrete time, discrete valued recursive networks.

Contents: Principles, problems, and approaches; deterministic and statistical approaches; thermodynamic extension; higher order networks; network design.